Reporting Measured Numbers (Significant Digits)

Reporting a measured number with an appropriate number of digits is important since any number published in a scientific report implicitly implies a certain experimental resolution. In other words, if you report a length of 1.045 mm, you are not just asserting the length, but also that your measuring device had a resolution of thousandths of a millimeter. So, if you display too many digits, you are making a misleading claim about the precision of your measurement.

Deciding how many digits are appropriate to report is not always simple - the Physics and Astronomy departments covers the proper mathematical theory on this topic later courses PHYS 241 and PHYS 245. However, there is a very simple *approximate* way to do it that works quite well: the method of "significant digits." Keeping track of the number of significant digits is quite easy to do and gets the number of digits about right (± 1 digit), which keeps the experimental uncertainty to within about an order of magnitude (i.e. a factor of ten) of the correct value. Because it is simple and approximately right, the method is very worthwhile and all serious science students should get in the habit of keeping track of the number of significant digits in every reported number. Significant digits turns out to be both a useful and easy way to approximate our uncertainty.

Determination of the Number of Significant Digits in a Measured Quantity:

The number of significant digits in a measured number is determined by the resolution of the measuring equipment and by the magnitude of the quantity measured.

Examples:

A ruler with ± 0.1 mm precision is used to measure a one millimeter long sample.
The result is: 1.0 mm $\pm$ 0.1 mm. (The quantity 1.0 mm has 2 significant digits).

The same ruler is used to measure a ten-centimeter long sample.
The result is: 100.0 mm $\pm$ 0.1 mm (The quantity 100.0 mm has 4 significant digits).

Vernier calipers with $\pm$0.05 mm precision are used to measure the one millimeter sample.
The result is: 1.00 mm $\pm$ 0.05 mm (The quantity 1.00 mm has 3 significant digits).

(Note that the uncertainty has only one significant digit – uncertainties are not accurately known quantities. Note also that the measured number has as many digits after the decimal point as the uncertainty so that they can be added.)

Propagation of Significant Digits in a Calculation:

1. Adding/Subtracting

   The number of significant digits in the result is determined by the operand with the least number of significant digits after the decimal point.
   Examples:

$$\begin{array}{r} 2.005 \\ 0.04 \\ +13.2415 \\ \hline 15.29 \end{array}$$

$$\begin{array}{r} 1.954 \\ -0.43 \\ \hline 1.52 \end{array}$$

2. Multiplying/Dividing or Other Operations (exponeniating, etc.)

   The number of significant digits in the result is the same as the smallest number of significant digits in any of the operands.
   Examples:

   $2.005 \times 1.04 \times (3.2 \times 10^2) = 6.7 \times 10^2$

   (Note: Writing the above result as 670 *is ok, but it is somewhat ambiguous whether or not the zero is supposed to be significant. The use of scientific notation avoids this ambiguity).*

   $\sin(1.2\,\pi) = -0.59$

3. Using extra digits in intermediate steps of a calculation to avoid round-off error.

   You may use an extra digit in intermediate steps to avoid error due to repeated round-offs. (Note that one or at most two extra digits is plenty; using all eight digits displayed by the calculator is always a waste of time).

   If you use extra digits, this should be done in a part of your report designated for calculations. **In the section of your report where you provide answers to the questions, the reported numerical values must have an appropriate number of significant digits.**