Physics 167 – Astronomy

Homework #7

Chapter 13

1. Isolated molecular clouds can have a temperature as low as 10 K and a particle density as great as 100,000 particles per cubic centimeter. What is the minimum mass that a cloud with these properties needs in order to form a star?

2. Near the end of its life, the Sun's radius will extend nearly to Earth's orbit. Estimate the volume of the Sun at that time. How does that density compare with the density of water (1 g/cm^3)? How does it compare with the density of Earth's atmosphere at sea level (about 10^{-3} g/cm^3)?

3. The distance from Earth of the red supergiant Betelgeuse is approximately 643 light-years. If it were to explode as a supernova, it would be one of the brightest stars in the sky. Right now, the brightest star other than the Sun is Sirius, with a luminosity of $26L_{Sun}$ and a distance of 8.6 light-years. How much brighter in our sky than Sirius would the Betelgeuse supernova be if it reached a maximum luminosity of $10^{10}L_{Sun}$?

4. Using the periodic table (Appendix D), determine which elements are made by the following nuclear fusion reactions. (You can assume that the total number of protons in the reaction remains constant.)
a. Fusion of a carbon nucleus with another carbon nucleus.
b. Fusion of a carbon nucleus with a neon nucleus.
c. Fusion of an iron nucleus with a helium nucleus.

5. The Algol binary system consists of a $3.7M_{Sun}$ star and a $0.8M_{Sun}$ star with an orbital period of 2.87 days. Use Newton's version of Kepler's third law to calculate the orbital separation of the system. How does that separation compare with the typical size of a red giant star?

Chapter 14

6. Calculate the Schwarzschild radius in kilometers for each of the following:
a. 10^8M_{Sun} black hole in the center of a quasar
b. $5M_{Sun}$ black hole that formed in the supernova of a massive star
c. a mini-black hole with the mass of the Moon
d. a mini-black hole formed when a super-advanced civilization decides to punish you (unfairly) by squeezing you until you become so small that you disappear inside your own event horizon

7. Theoretical models of the slowing of pulsars predict that the age of a pulsar is approximately equal to $p/(2r)$, where p is the pulsar's current period and r is the rate at which the period is slowing with time. Observations of the pulsar in the Crab Nebula

show that it pulses 30 times per second, so $p = 0.0333$ second, but the time interval between pulses is growing longer by 4.2×10^{-13} second with each passing second, so $r = 4.2 \times 10^{-13}$ second per second. Using that information, estimate the age of the Crab Nebula pulsar. How does your estimate compare with the true age of the pulsar, which was born in the supernova observed in A.D. 1054?

8. A clump of matter does not need to be extraordinarily dense in order to have an escape velocity greater than the speed of light, as long as its mass is large enough. You can use the formula for the Schwarzschild radius R_S to calculate the volume $\frac{4}{3}\pi R_S^3$ inside the event horizon of a black hole of mass *M*. What does the mass of a black hole need to be in order for its mass divided by its volume to be equal to the density of water ($1\ \mathrm{g/cm^3}$)?

9. In a massive star supernova explosion, a stellar core collapses and forms a neutron star roughly 10 km in radius. The gravitational potential energy released in such a collapse is approximately equal to $\frac{GM^2}{r}$, where *M* is the mass of the neutron star (assume $M = 1.5M_{Sun}$), *r* is its radius, and the gravitational constant is $G = 6.67 \times 10^{-11}$ $\mathrm{m^3.kg^{-1}.s^{-2}}$. Using this formula, estimate the amount of gravitational potential energy released in a massive star supernova explosion. How does it compare to the amount of energy released by the Sun during its entire main-sequence lifetime?

10. Models indicate that the gravitational wave signal shown in Figure 14.21 came from the merger of two black holes with masses of $29M_{Sun}$ and $36M_{Sun}$, and resulted in a single black hole with a mass of $62M_{Sun}$. The difference in total mass between the start and the finish of the merger corresponds to the amount of energy carried away in the form of gravitational waves. Use Einstein's formula $E = mc^2$ to calculate this amount of energy. Be sure to put the mass in units of kg and the speed of light in m/s, so that the resulting energy has units of joules. Compare your answer to the energy released by a supernova (in the previous problem) and to the energy released by the Sun during its entire main-sequence life.