Classical Dynamics – Physics 312 – Homework 14

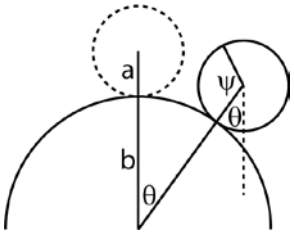
1. A flat circular disk of mass *m* and radius *b* is spun about its axis. If its density is proportional to the distance *r* from the origin, find its moment of inertia.

2. a. A "physical pendulum" consists of a rigid body of mass *m* that pivots about a fixed point. The body's moment of inertia about the pivot is *I*, and the distance from the pivot to the center of mass is *L*. Show that the period of small oscillations is $T = 2\pi\sqrt{\frac{I}{mgL}}$.

A circular hoop of radius *a* swings as a physical pendulum about a point on the circumference. Find the period of oscillation for small amplitude if the axis of rotation is
b. normal to the plane of the hoop, or
c. in the plane of the hoop.

3. A uniform solid ball (sphere) contains a hollow spherical cavity at its center, the radius of the cavity being ½ the radius of the ball. Show that the acceleration of the ball rolling down an incline is just $\frac{98}{101}$ of that of a uniform solid ball with no cavity.

4. A uniform circular cylinder of radius *a* is balanced on the top of a perfectly rough fixed cylinder of radius *b* ($b > a$), the axes of the two cylinders being parallel. If the balance is slightly disturbed, show that the rolling cylinder leaves the fixed one when the line of centers makes an angle with the vertical of $\cos^{-1}\left(\frac{4}{7}\right)$.



Hint: The condition for rolling without slipping can be expressed $a\psi = b\theta$ (see the figure). The angular velocity of the rolling cylinder, with the angle measured from the vertical direction, is $\omega = \dot{\psi} + \dot{\theta}$.