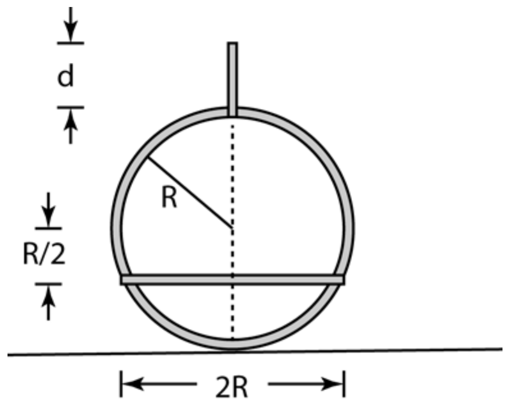
Physics 312 – Classical Mechanics – Homework 5



1. The structure shown above is made of wires soldered together, and it is placed on a flat tabletop and confined to move in a plane. The structure consists of a circle of radius R, a rod of length $2R$ passing through the midpoint of a radius, and a rod of length d. All wires have uniform density m/R.

a. Find the position of the CM (center of mass) measured from the bottom. Express your answer in the form of $R\ f(x)$ where $x = d/R$.

b. Find the gravitational potential energy as a function of angle θ if the structure is tipped by θ.

c. Find the condition on d/R needed for the structure to be in a stable equilibrium in the vertical position as it is shown in the picture.

2. A bead of mass m is free to slide along a frictionless wire bent in the curve $y = \frac{1}{a^2}x^3$ where a is a positive constant. The bead starts from rest at $x = a$ and slides under the influence of a constant gravitational field g pointing in the negative y direction. Find the time required for the bead to reach the origin. Express your answer in terms of the constants a and g.

Hint: Use the energy method. You may use Mathematica to numerically evaluate the integral you obtain (after making it dimensionless).

3. A particle of mass m is constrained to move without friction along the x-axis, subject to a potential energy given by $U(x) = U_0\left(\frac{1}{\sqrt{1-x^2/b^2}} - 1\right)$ where U_0 and b are positive constants. Show that for small oscillations about $x = 0$, the particle undergoes simple harmonic motion. What condition on x is required for the oscillations to be "small" (i.e. simple harmonic)? Find the period T of the oscillations.

4. A simple harmonic oscillator is started with initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$.

a. Find the solution in the form $x(t) = A\ \cos(\omega t + \phi)$. That is, find A and ϕ in terms of x_0 and v_0.

b. Find the solution in the form $x(t) = B\ \cos(\omega t) + C\ \sin(\omega t)$. That is, find B and C in terms of x_0 and v_0.

5. Taylor Problem 5.11. *Hint*: The energy conservation equation relates the x and v variables.