Physics 312 – Classical Mechanics – Homework 7

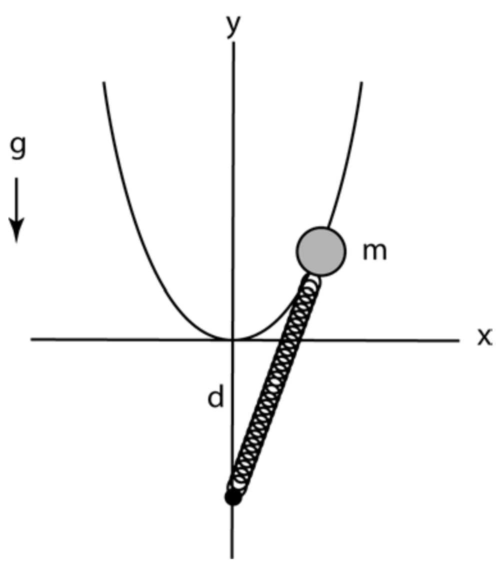
1. Consider a weakly-damped, driven SHO. Show that the driving frequency for which the steady-state amplitude is ½ the steady-state amplitude at the resonant frequency is given by $\omega \cong \omega_0 \pm \beta\sqrt{3}$.

2. A mass moves along the *x*-axis subject to an attractive force $17c^2mx/2$ and a retarding force $3cm\dot{x}$, where *x* is its distance from the origin and *c* is a constant. A driving force $mA\cos\omega t$ is applied to the particle along the *x*-axis (*A* = constant).
a. What value of ω results in steady-state oscillations about the origin with maximum amplitude?
b. What is the maximum amplitude?
Express your answers in simplest form.

3. A damped harmonic oscillator is driven by an external force of the form $mf_0 \sin\omega t$. The equation of motion is therefore $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0 \sin\omega t$. Carefully explaining all steps, show that the steady-state solution is given by

$$x(t) = A(\omega)\sin(\omega t - \delta).$$

Find $A(\omega)$ and $\delta(\omega)$.



4. A bead of mass *m* slides on a frictionless wire bent into the shape of a parabola

$$y = \frac{1}{d}x^2$$

as shown above. Gravity acts in the negative *y* direction. A spring with elastic constant k and rest length $d/2$ connects the bead to a fixed anchor at the point $(0, -d)$. Find the frequency ω of small oscillations about equilibrium.

Hint: Find the potential energy *U* of the bead. Then expand *U* in series, keeping only the leading x^2 term, to obtain $U \cong \frac{1}{2}k_{eff}x^2$. Knowing k_{eff}, it is easy to find ω.