Classical Mechanics – Physics 312 – Homework 8

1. Find the function $y(x)$ that minimizes the integral $\int x\sqrt{1-y'^2}\,dx$. Your answer will contain two arbitrary constants.
Hint: The following result may be helpful: $\frac{d}{dx}\sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$.

2. Taylor 6.20. Be sure to add a few words of explanation to your calculation.

3. Taylor 6.22.
Hint: You can evaluate the arbitrary integration constants by using the boundary conditions that $y = 0$ when $s = 0$ and when $s = \ell$. To show that the curve is a semicircle, it is easiest to express the equation of the semicircle in *x*-*y* coordinates and check if $x(s)$ and $y(s)$ satisfy it. You'll need to find $x(s)$, which you can do using the relation $ds^2 = dx^2 + dy^2$ once you have found $y(s)$.

4. Using Fermat's principle, find the path followed by a light ray if the index of refraction is proportional to $\sqrt{y}$. Solve for $y(x)$ (there will be two undetermined constants of integration) and describe the curve (i.e. it has a name).
Hint: The integrand has no explicit x dependence, so the result of Problem 3 will be helpful.

5. Taylor 6.24.
Hint: You should obtain a relation of the form $r = c\ \sin(\phi - \phi_0)$ where *c* and ϕ_0 are constants obtained in the integration. It is convenient to note that one may take $\phi_0 = 0$ without loss of generality, since this merely amounts to choosing the direction of the *x*-axis. Then one can multiply the relation $r = c\ \sin\phi$ by *r*, express in *x*-*y* coordinates, and complete the square to obtain the desired equation of a circle.