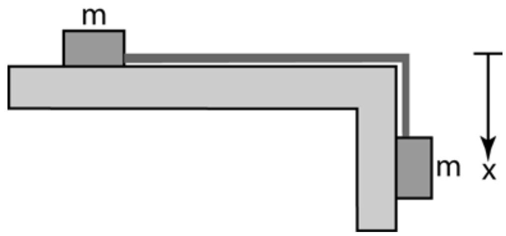
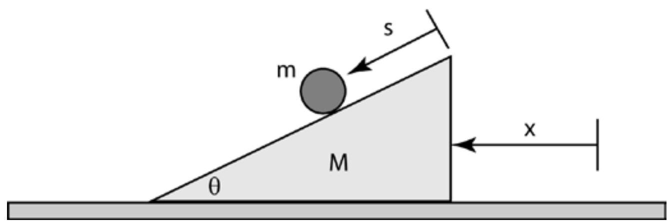
Classical Mechanics – Physics 312 – Homework 9



1. A block of mass m is placed on a frictionless horizontal table. It is connected to an identical block by a thin flexible chain of mass m and length L which runs over the edge of the table as shown in the picture above. The system starts at rest with one block just barely hanging over the edge of the table.

a. Find the acceleration of the system. *Hint*: Use the variable x shown in the picture as the coordinate to describe the system, and write the Euler-Lagrange equation.

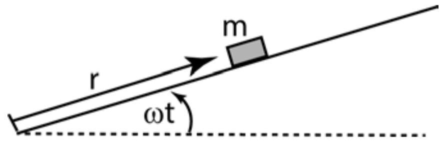
b. Solve the differential equation to find $x(t)$. Remember that the solution is the sum of the solution to the homogeneous equation and a particular solution to the inhomogeneous equation. Since the inhomogeneous term is a constant, seek a trial particular solution of the form $x(t) = c$ and find which value of the constant c solves the equation.



2. A solid spherical ball of mass m rolls down a movable wedge of mass M. The wedge makes angle θ with the horizontal, and it is free to slide on a frictionless horizontal table as shown above. The ball rolls without slipping on the wedge. Find the acceleration of the wedge.

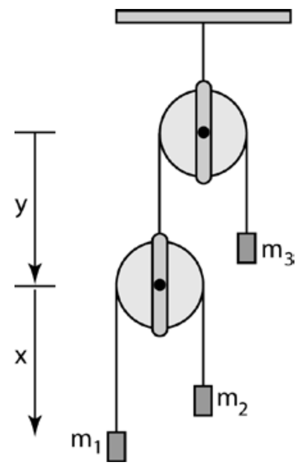
Hint: Use the coordinates x and s shown in the diagram. Note that s measures the position of the ball from the top of the incline (not an inertial frame), so be careful to evaluate the kinetic energy of the ball with respect to an inertial (fixed) frame. Also recall that the kinetic energy of the ball is given by $T = \frac{1}{2} m\, v_{cm}^2 + \frac{1}{2}\, I_{cm} \omega^2$, and that the moment of inertia of a solid sphere is $I_{cm} = \frac{2}{5} m\, R^2$.

(over)



3. A bead of mass m slides on a smooth wire whose angle ϕ with the *x*-axis is increasing at constant rate ω. The wire is rotating in the horizontal plane, so gravity plays no role in this problem. The bead starts from rest at distance L from the origin at time $t = 0$, when the angle $\phi = 0$. Find the radial position $r(t)$ of the bead as a function of time.

Hint: It is convenient to use polar coordinates. You'll need to solve the Euler-Lagrange differential equation for $r(t)$.



4. An Atwood machine (with masses m_1 and m_2 hanging over a massless pulley) is suspended over another massless pulley, with a mass m_3 connected on the other side (see the picture). Assume m_1 = 1 kg, m_2 = 2 kg, and m_3 = 4 kg. Find the acceleration of mass m_1 with respect to the ceiling.

Hint: Use the coordinates *x* and *y* shown in the picture. Be careful to evaluate the kinetic and potential energies in an inertial (fixed) frame. You'll need to obtain the Euler-Lagrange equations for *x* and *y*.