Physics 312 – Classical Mechanics – Homework #1

1. For what value or values of q is the vector $\vec{A} = q\,\hat{x} + 3\,\hat{y} + \hat{z}$ perpendicular to the vector $\vec{B} = q\,\hat{x} - q\,\hat{y} + 2\,\hat{z}$?

2. a. If an object's position is given by $\vec{r}(t) = 5\cos(\pi t^2)\hat{x} + t^3\hat{y}$, find the speed of the object at $t = 1$ sec.
b. Is the object speeding up or slowing down at $t = 1$ sec?

3. a. Prove that $\vec{v} \cdot \vec{a} = v\,\dot{v}$.
b. Explain why the above means that if a particle's speed is constant, its velocity and acceleration vector are perpendicular.
Hint: Differentiate both sides of the equation $\vec{v} \cdot \vec{v} = v^2$. Note that $\dot{v}$ is not the same as $|\vec{a}|$; $\dot{v}$ is the magnitude of the acceleration of the particle along its instantaneous direction of motion.

4. a. A buzzing fly moves in a helical path given by the equation

$$\vec{r}(t) = b\,\sin\omega t\;\hat{x} + b\,\cos\omega t\;\hat{y} + c\,t^2\,\hat{z}.$$

Show that the magnitude of the acceleration of the fly is constant, provided b, ω, and c are constant.
b. Repeat the above question using cylindrical coordinates where $r = b$, $\phi = \omega t$, and $z = c\,t^2$.

1.18 ** The three vectors **a**, **b**, **c** are the three sides of the triangle ABC with angles α, β, γ as shown in Figure 1.15. **(a)** Prove that the area of the triangle is given by any one of these three expressions:

$$\text{area} = \tfrac{1}{2}|\mathbf{a}\times\mathbf{b}| = \tfrac{1}{2}|\mathbf{b}\times\mathbf{c}| = \tfrac{1}{2}|\mathbf{c}\times\mathbf{a}|.$$

(b) Use the equality of these three expressions to prove the so-called law of sines, that

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$

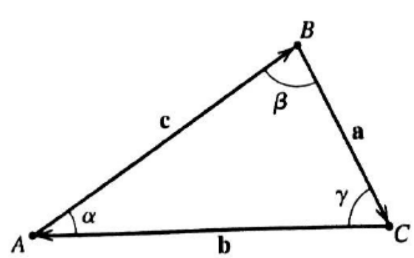

Figure 1.15 Triangle for Problem 1.18.

1.22 ★★ The two vectors **a** and **b** lie in the xy plane and make angles α and β with the x axis. **(a)** By evaluating $\mathbf{a} \cdot \mathbf{b}$ in two ways [namely using (1.6) and (1.7)] prove the well-known trig identity

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

(b) By similarly evaluating $\mathbf{a} \times \mathbf{b}$ prove that

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$$