Physics 312 – Classical Mechanics – Homework #2

1. The velocity of a particle in rectilinear motion varies with displacement *x* according to the relation

$$\dot{x} = b\, x^{-3},$$

where b is a positive constant. Find the force acting on the particle as a function of *x*. (Hint: $= m\ddot{x} = m\, \dot{x}\, \frac{d\dot{x}}{dx}$)

2. Find the velocity and position as functions of time for a particle of mass *m* subject to the force given below and starting with the given initial conditions. from rest at $x = 0$ and $t = 0$, subject to the force given by:
a. $F_x = F_0 + ct$, starts from rest at $x = 0$ and $t = 0$.
b. $F_x = cx^{-1/2}$, starts from rest at $x = 0$ at $t = 0$, where F_0, *c*, and *a* are constant.

3. A heavy block of mass *m* slides on a horizontal surface coated with oil so that the block experiences a viscous drag force given by

$$F(v) = -c\, v^{3/2}.$$

If the initial speed of the block is v_0 at $x = 0$, show that the maximum distance the block can travel is $2mv_0^{1/2}/c$.

4. A pistol is fired either straight upward or straight downward from the origin at $t = 0$. Assuming that the air drag on the bullet varies quadratically with speed, show that the speed varies with height according to the relations
$v^2 = A\, e^{-2kx} - \frac{g}{k}$ (upward motion)
$v^2 = \frac{g}{k} - B\, e^{2kx}$ (downward motion)
where $k = c/m$, where *c* is the drag constant, *g* is the acceleration due to gravity, and *A* and *B* are constants. Derive expressions for *A* and *B* in terms of the given constants and the initial velocity v_0 is the initial velocity of the bullet.

5. A bead of mass *m* is free to slide on a frictionless wire bent in the shape of a cosine curve $y = a\ \cos\left(\frac{x}{b}\right)$, where *a* and *b* are constant. Gravity points in the negative *y* direction. Suppose the bead starts at rest at the top of a peak.
a. Find the radius of curvature of the point at the bottom of a trough.
b. Find the tangential and normal components of the acceleration of the bead at the bottom of a trough.
c. Find the constraint force (normal force) vector exerted by the wire on the bead at the bottom of a trough.