Physics 312 – Classical Mechanics – Homework #3

1. A rocket, whose total mass is m_0, contains a quantity of fuel, whose mass is ϵm_0 (where $0 < \epsilon < 1$). Suppose that, on ignition, the fuel is burned at a constant mass-rate k, ejecting gasses with a constant speed u_0 relative to the rocket. Assume the rocket is in a force-free environment and starts from rest.
a. Find the distance that the rocket has traveled at the moment it has burnt all the fuel.
Hint: We are given $\frac{dm}{dt} = -k$, so integrating with respect to time gives $m(t) = m_0 - kt$. Solve the rocket equation for $\frac{dv}{dt}$, and integrate with respect to time to find $x(t)$. You may use Mathematica to evaluate the integral you get, after first making a variable change to make the integral dimensionless.

b. What is the maximum possible distance that the rocket can travel during the burning phase?
Hint: Find the limit $\lim_{\epsilon \to 1} x$.

2. A rocket traveling through the atmosphere experiences a linear air resistance $-c\vec{v}$. Find the differential equation of motion when all other external forces are negligible. Integrate the equation and show that if the rocket starts from rest, the final speed is given by

$$v = u_0 \alpha \left[1 - \left(\frac{m}{m_0} \right)^{1/\alpha} \right]$$

where u_0 is the relative speed of the exhaust fuel, $\alpha = |\dot{m}/c|$ is constant, m_0 is the initial mass of the rocket plus fuel, and *m* is the final mass of the rocket.

Hint: Be careful with the signs: u_0 points in the opposite direction as *v*, and $\dot{m} < 0$. It will be helpful to use the chain rule to write $\frac{dv}{dt} = \frac{dv}{dm}\frac{dm}{dt} = \frac{dv}{dm}\dot{m} = -\frac{dv}{dm}c\alpha$.

3. Show that the kinetic energy of a two-particle system is given by

$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}\mu v^2,$$

where $m = m_1 + m_2$, *v* is the relative speed (the speed of one of the particles with respect to the other), and μ is the reduced mass, given by $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$.

Hint: Start with $T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$. Then make a change of variables $v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$, $v = v_1 - v_2$. Using this change of variables, you can replace the variables v_1 and v_2 with the variables v and v_{cm}.