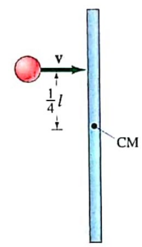
Physics 312 – Classical Mechanics – Homework #4

1. Show that the angular momentum of a two-particle system is given by

$$\vec{r}_{cm} \times m\vec{v}_{cm} + \vec{r} \times \mu\vec{v}$$

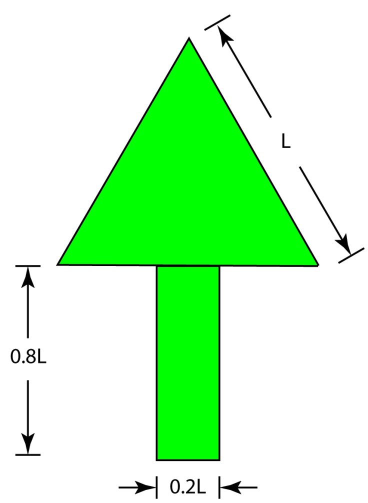
where $m = m_1 + m_2$, $\vec{v}$ is the relative velocity (the velocity of one of the particles with respect to the other), $\vec{r}$ is the relative position, and μ is the reduced mass.



2. A thin rod of mass *m* and length ℓ rests on a frictionless table and is struck at a point $\ell/4$ from its center of mass by a clay ball of mass m moving at speed *v*. The ball sticks to the rod.

a. If the rod is free to pivot about a frictionless pin at its center, find the angular velocity of the rod after the collision. Also find the velocity of the point at the top of the rod just after the collision.

b. Find the quantity of thermal energy produced in the collision.



3. Find the distance of the CM from the base of the “pine tree” shape shown above. The shape is cut from a thin layer of material with uniform density. The triangle is equilateral.

4. A particle of mass *m* = 2 kg moves under the potential energy function $U(x, y, z) = (kx + 2\,k^2y^2 + 3\,k^3z^3)$ where *k* = 1 N.

a. Suppose the particle has speed $v_0 = 3$ m/s when it passes through the origin. What will its speed be if and when it passes through the point (1,1,1)?

b. Suppose the particle's speed v_0 at the origin is not known and that the point (1,1,1) is a turning point of the motion (a point where $v = 0$). What must v_0 be?
c. Write down the differential equations describing the particle's motion (the *x*, *y*, and *z* component equations obtained from $\vec{F} = m\vec{a}$). You do not need to solve the equations.

5. a. Show that the variation of gravity with height is given approximately by

$$U = mgz\left(1 - \frac{z}{R_e}\right)$$

where R_e is the radius of the earth and *z* is the height above the surface. Find the force derived from the above potential function.
Hint: Start with $U = V(R_e + z) - V(R_e)$ with $V(r) = -\frac{G\, m\, M_e}{r}$, where *G* is the universal gravitation constant and M_e is the mass of the earth. Approximate using a Taylor series.

b. For an object dropped from initial height h above the surface of the earth, find its speed when it hits the ground to first order in $\frac{h}{R_e}$. If the object is dropped from a height of 100 m, by what percent does the object's speed differ from the speed one calculates assuming the gravitational force is constant?